

# UNIVERSITÀ DEGLI STUDI DI PALERMO

DEPARTMENT	Matematica e Informatica
ACADEMIC YEAR	2023/2024
MASTER'S DEGREE (MSC)	MATHEMATICS
INTEGRATED COURSE	SUPERIOR ANALYSIS
CODE	07799
MODULES	Yes
NUMBER OF MODULES	2
SCIENTIFIC SECTOR(S)	MAT/05
HEAD PROFESSOR(S)	TRAPANI CAMILLO Professore Ordinario Univ. di PALERMO
OTHER PROFESSOR(S)	TRAPANI CAMILLO Professore Ordinario Univ. di PALERMO
	TORNATORE Professore Associato Univ. di PALERMO ELISABETTA
CREDITS	12
PROPAEDEUTICAL SUBJECTS	
MUTUALIZATION	
YEAR	1
TERM (SEMESTER)	Annual
ATTENDANCE	Not mandatory
EVALUATION	Out of 30
TEACHER OFFICE HOURS	TORNATORE ELISABETTA
	Tuesday 9:30 10:30 Studio del docente
	TRAPANI CAMILLO
	Wednesday 14:30 16:30 Studio del docente: Dipartimento di Matematica e Informatica, Via Archirafi 34, 1° Piano, Ufficio 115b

PREREQUISITES	Differential and integral calculus in one or more variables; Lebesgue measure and integral; Basic aspects of the theory of functions of complex variables.
LEARNING OUTCOMES	Knowledge and understanding: Acquisition and ability 'to use the ideas and methodologies of the spaces with inner product and normed spaces, Banach algebras and C *-algebras, the series of Neumann, the spectrum and spectral radius, algebra functions, the C algebra of bounded operators.  Acquisition of advanced tools for the understanding of recent research articles in Functional analysis and Non-commutative Analysis. Ability to use their own specific languages of these disciplines and produce personal and original scripts in research mathematics and its applications.
	Applying knowledge and understanding: Ability to recognize, and organize autonomously, the necessary elements for a deep study of a recent research article in Functional Analysis and Analysis Nor commutative. Ability to formalize mathematically problems and develop proofusing techniques inspired by consolidated mathematical literature. The verification of the ability 'as acquired is done through active participation of the student to the resolution of problems and issues.
	Making judgments: The student should be able to evaluate the implications and the results contained in a recent research article in Functional Analysis and Analysis Nor commutative. The full understanding of the fundamental concepts and key techniques introduced in the course allow 'the student to acquire the ability' to critically analyze scientific texts and to be able to evaluate the content and implications of the results in a recent research article in Functional Analysis Analysis and non-commutative.
	Communication skills: Ability to clearly present the main topics of the course. Capacity to expose the results contained in a recent research article in a functional analysis or in nor commutative analysis analysis. In order to check communication skills students will be involved in seminar activities.  Learning skills: Ability to upgrade knowledges with the reading of scientific publications Capacity 'to follow, using the knowledge acquired in the course, both advancer courses and research seminars in the field Functional Analysis or Nor commutative Analysis.
ASSESSMENT METHODS	The final exam consists of an oral interview of general character on course arguments and in the discussion of a pre-assigned topic. In-course tests, if passed, may be the substitute for the general interview.  The purpose is to assess, not only the knowledges of the candidate and his ability to apply them, but also the possession of an appropriate language and the reached degree of mathematical rigor.  The final assessment, properly graded, will be made on the basis of following conditions:  a) Basic knowledge of the topics proposed and limited capacity to apply them independently; sufficient capacity to carry out a rigorous reasoning and sufficient command of the language (18-21 rating); b) Fairly good knowledge of the topics proposed and sufficient capacity to apply them independently; fairly good ability to complete a rigorous reasoning and good appropriate language (22-25 rating); c) Good knowledge of the topics proposed and fairly good capacity to apply them independently; good ability to complete a rigorous reasoning; good command of the appropriate language (26-28 rating); d)Very good and extensive knowledge of the topics proposed; ability to apply them with mathematical rigor and independently; possess of very good communication skills(29-30L rating).
TEACHING METHODS	Lectures and other didactical activities last the whole year but take place in two educational periods according to the schedule of classes.  The achievement of learning outcomes will be reached through lectures, exercises, homework. The students will be also asked to give short seminars or topics of related to the lectures or on complementary arguments. In both modules in-course tests will be scheduled.

## MODULE NON COMMUTATIVE ANALYSIS

Prof. CAMILLO TRAPANI

#### SUGGESTED BIBLIOGRAPHY

Testo di riferimento: C. Trapani, Teoria degli Operatori, dispensa on-line

Letture consigliate:

W. Rudin, Functional Analysis, McGraw-Hill, 1991

M. Reed, B. Simon, Functional Analysis, Academic press, 1981

G.K.Pedersen, Analysis now, Springer,2012

AMBIT	50398-Formazione teorica avanzata
INDIVIDUAL STUDY (Hrs)	94
COURSE ACTIVITY (Hrs)	56

#### **EDUCATIONAL OBJECTIVES OF THE MODULE**

Acquisition of concepts and methods of non-commutative analysis (Banach algebras and C \* -algebras) and of the fundamental aspects of the theory of operators in Hilbert spaces; development of the capacity to apply them in other areas of mathematics.

## **SYLLABUS**

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Hrs	Frontal teaching
6	Banach algebras and C* -algebras, Ideals and quotients. Invertible elements. Neumann series. Spectrum and spectral radius. Mazur theorem.
6	Characters and maximal ideals. Algebras of functions. Gelfand transform. Characterization of commutative C $^{\star}$ -algebras.
4	Representations, positive functionals, Gelfand-Naimark theorem
5	The C*-algebra of bounded operators. Symmetric and unitary operators; projections. Spectrum of an operator and its classification.
5	Compact operators. Riesz-Schauder theorem. Canonical form of compact operators
6	Unbounded operators and their spectra
Hrs	Practice
3	Examples, exercises and additional proofs on Banach algebras
4	Examples, exercises and additional proofs on commutative Banach algebras, characters, Gelfand transform
3	Examples, exercises and additional proofs on C*-algebras
4	Exercises on bounded operators
3	Exercises on the spectrum of bounded operators
3	Exercises on compact operators

### **MODULE FUNCTIONAL ANALYSIS**

Prof.ssa ELISABETTA TORNATORE

#### SUGGESTED BIBLIOGRAPHY

- H. Brezis, Analisi Funzionale- Liguori Editore(ISBN:9788820715014) W. Rudin, Functional Analysis -McGraw Hill (ISBN: 9780070542365)
- D. Averna, Analisi Funzionale Spazi di Banach e spazi di Hilbert (dispenze disponibili all'indirizzo http://math.unipa.it/averna/)

AMBIT	50398-Formazione teorica avanzata
INDIVIDUAL STUDY (Hrs)	94
COURSE ACTIVITY (Hrs)	56

## **EDUCATIONAL OBJECTIVES OF THE MODULE**

Illustrate the fundamental concepts of Hilbert spaces and Banach spaces.

#### **SYLLABUS**

Hrs	Frontal teaching	
8	Hilbert Spaces: Pre-Hilbert spaces, the Hilbert space I^2, the Hilbert space L^2.	
8	Geometry Hilbert spaces: subspaces, orthogonal subspaces, bases, isomorphism. Fourier series in Hilbert space.	
8	Linear bounded operators: Linear and bounded applications, linear operators, bilinear forms, adjoint operators, projection operator.	
8	Banach spaces: normed linear spaces, linear operators, linear functional, operators and linear functional on finite dimensional spaces. space normed of operators, dual spaces, the theorems of Hahn-Banach, reflexive spaces, theorem of category and uniformly bounded- strong and weak convergence.	
Hrs	Practice	
8	Examples, exercises and additional proofs on subspaces of Hilbert spaces.	
8	examples, exercises and additional proofs on linear operators.	
8	examples, exercises and additional proofs on normed linear spaces	