## UNIVERSITÀ DEGLI STUDI DI PALERMO



| PREREQUISITES | Combinatorics. <br> Analytic geometry. <br> Linear systems. <br> Set theory. <br> Sequences and series. <br> Differential and integral calculus. <br> Complex numbers. |
| :---: | :---: |
| LEARNING OUTCOMES | 1) KNOWLEDGE AND UNDERSTANDING <br> The student should know the following topics: <br> - Elements of logic and combinatorics; <br> - Various concepts of probability <br> - Setting of Coherence <br> - Elementary Probability; <br> - Events and conditional events; <br> - Random quantities and probability distributions; <br> - Summary statistics; <br> - Classical problems of probability. <br> - Random vectors, joint and marginal probability distributions; <br> - Functions of random quantities and of random vectors; <br> - Relations among random quantities; <br> - Some limit theorems. <br> 2) APPLYING KNOWLEDGE AND UNDERSTANDING <br> The student should know how to exploit probabilistic tools to reasoning under uncertainty. In particular, the student should be able to: <br> - Describe the uncertainty; <br> - Check the coherence of a probability assessment on an arbitrary finite family of events; <br> - Apply Bayes' rule and compound probability theorem; <br> - Solve classical problems of probability; <br> - Select the suitable probability distribution to describe some standard random problems; <br> - Apply limit theorems; <br> - Compute marginal probability distributions from joint probability distributions. <br> - Solve problems involving functions of random quantities. <br> 3) MAKING JUDGEMENTS <br> Being able to motivate the choice of some probabilistic tools and models in order to properly study some problems under partial or incomplete knowledge. <br> 4)COMMUNICATION. <br> The student should know how to illustrate in a clear and coherent way the description and the analysis of a random problem to both expert and non-expert people. <br> 5) LIFELONG LEARNING SKILLS. <br> Being able to exploit the power of self-discovery, exploration, learning and mastery. In particular the student should be able to deep some well-known notions in probability theory and to discover new notions by studying from other research books or from research articles. |
| ASSESSMENT METHODS | The exam is composed by a written test (or two intermediate written tests) followed by an oral test <br> Written tests <br> The written test will be composed of 4 exercises: 2 exercises which manly concern topics of the first half of the course and <br> 2 exercises which manly concern topics of the second half of the course. A mark of 7.5 points will be given for each exercise correctly solved. The evaluation of the written test will be made by means of a 30-point scale. Candidates who do not pass the written test are encouraged to repeat the written test. <br> Intermediate written tests. <br> The first intermediate written test will be composed of 2 exercises which mainly concern topics of the first half of the course. <br> The second intermediate written test will be composed of 2 exercises which mainly concern topics of the second half of the course. <br> A mark of 15 points will be given for each exercise correctly solved. <br> The evaluation of the intermediate written test will be made by means of a 30point scale. Those who pass both intermediate written tests will avoid the written test and will receive as score of the written test the average of the two intermediate scores. <br> Some solved tests will be made available during the lectures. <br> The written tests are designed in order to evaluate the degree of knowledge and the ability of the candidate to solve problems similar to those illustrated during |


|  | the lectures and during practicals. <br> In particular, through them it will be possible to measure the ability of the candidate in order to: <br> formalize a random phenomenon; <br> motivate the choice of a probabilistic model and of some probability distributions; provide summary statistics; <br> understand conditionalization; <br> apply or re-elaborate theoretical results; apply limit results; <br> exploit standard normal table; <br> apply mathematical tools to solve random problems. <br> Oral Test <br> By starting from a deepening on the written test, the oral test, through the formulation of some questions, will be used to <br> evaluate the degree of knowledge of the student in some theoretical results and in the use of technical language. The score of the oral test will be a number usually belonging to the set $\{-4,-3,-2,-1,0,1,2,3,4\}$. The score of the oral test will be used to finalize and / or improve the grade obtained in the written tests. Finale grade <br> By setting S="the score of the written test" and O="the score of the oral test", the final grade $V$ usually will be $\mathrm{V}=\mathrm{min}(\mathrm{S}+\mathrm{O}, 30)$, if $\mathrm{S}+\mathrm{O}>0$ or $\mathrm{V}=0$ otherwise. <br> The student pass the exam if the final grade is greater or equal than 18/30 <br> The minimum grade 18/30 in order to pass the exam will be given to the student that knows the basic fundamentals of the course, that is able to do some links among the different topics and that is able to use technical language. <br> The maximum grade $30 / 30$ cum laude will be given to the student that fully reaches the learning outcomes and that gets a score S+O greater than 30. |
| :---: | :---: |
| EDUCATIONAL OBJECTIVES | Being able to describe and represent some theoretical and real random problems through suitable probabilistic models, by eventually using R (or Python). In particular, the student should be able to: -motivate the choice of some probability distributions; provide the summary statistics; <br> -properly assess degrees of believe to (conditional or simple) events; update probability; <br> -exploit theoretical results for the study and analysis of random vectors; -reason and make decisions under uncertainty and/or partial knowledge <br> The student should also know how to solve some "classical paradoxes" of human reasoning by means of the elementary notions of probability. |
| TEACHING METHODS | Lectures and practicals. Usually, theory will be explained in the lecture and then applied and tested in the practical class. An english paper on probability or a book chapter will be shared with the students. |
| SUGGESTED BIBLIOGRAPHY | Testi di riferimento (References) <br> - [1] Sheldon Ross; Calcolo delle Probabilita' 3a ed.; Apogeo Education Maggioli Editore, 2013; ISBN 9788838788604 <br> (English version: A first course in Probability, 8th edition, Pearson) <br> - [2] Romano Scozzafava; Incertezza e Probabilita'; Zanichelli, 2003; ISBN: 9788808079756 <br> Dispense <br> Materiale didattico e soluzioni dei compiti di esami forniti dal docente (Notes and exercises provided by the Teacher). <br> Per alcuni argomenti del programma dell'insegnamento, non presenti nei testi consigliati, vengono forniti i seguenti riferimenti bibliografici. <br> (For some particular topics of the syllabus proper references are given below). <br> - [3] Paolo Baldi; Calcolo delle Probabilita'; McGraw-Hill, 2011, ISBN 978-8838666957; <br> - [4] Bruno de Finetti; Teoria delle Probabilita'; Giuffre', 2005 (ristampa), ISBN 978-8832821758; <br> (English version of the book: Theory of Probability: A critical introductory treatment, Wiley, 2017, ISBN 978-1119286370) <br> - [5] Luciano Daboni; Calcolo delle Probabilita' ed Elementi di Statistica; Utet, 1981, ISBN 978-8877502087. <br> - [6] Giorgio Dall'Aglio; Calcolo delle Probabilita'; Zanichelli, 2001, ISBN: 9788808176769. <br> - [7] Giuseppe Espa,Rocco Micciolo, Problemi ed esperimenti di statistica con R, Apogeo, 2014, ISBN 978-8838786105. <br> - [8] Kevin Ross \& Dennis L. Sun (2019) Symbulate: Simulation in the Language of Probability, Journal of Statistics Education, 27:1, 12-28, DOI: 10.1080/10691898.2019.1600387 <br> -[9] Daphne Koller, Nir Friedman, Probabilistic Graphical Models: Principles and Techniques, 2009, 978-0262013192 |

## SYLLABUS

| Hrs | Frontal teaching |
| :---: | :---: |
| 3 | Historical notes. Chevalier de Mere's Problem ([6]). Events, indicators, logical operations and relations ([2,5]). De Morgan's Law. Basic notions on combinatorial Analysis. Binomial theorem. Finite partition of the sure event. The classical interpretation of probability. Basic properties of probability. |
| 2 | The different approaches to probability: classical, axiomatic ([3]), frequentist, subjective ([2]). Betting criterion and coherence principle ([2]). Characterization of coherence. |
| 2 | Conditional events and conditional probability [2]. Compound probability theorem. Disintegration law. Bayes' rule. Events which are regarded stochastically independent. |
| 5 | Simple Random quantities [2]. Urn problems. Extraction of white balls, given $n$ draws with and without replacement in an urn with a known number of black and white balls. Binomial distribution. Hypergeometric distribution. Mixture of binomial distributions [2]. Mixture of hypergeometric distributions. |
| 4 | Discrete probability: properties of expectation and variance, cumulative distribution. Poisson and geometric distribution. Property of memoryless of the geometric distribution. Pascal and negative binomial distribution [2]. Markov's inequality. Chebyshev's inequality. |
| 5 | Continuous random quantities. Density [2,3], cumulative distribution function, expectation, variance. Property of continuous random quantities. Some continuous distributions: Uniform, Exponential, Normal, Gamma, Beta, etc. Hazard rate function [1,2]. Survival function. Memoryless proprerty of exponential distribution. |
| 5 | Discrete and continuous random vectors. Joint distribution functions, marginal distributions, conditional distributions. Independence. Covariance. Pearson correlation coefficient. Covariance matrix. |
| 3 | Characteristic function [2,3,6] and properties. Sum of random quantities. General Chi-square distribution. |
| 3 | Central limit theorem. Normal approximation of binomial distribution. Weak law of large numbers. |
| 4 | A short introduction to stochastic processes and to Poisson process. Markov chain. Classification of states. Transition matrix. Chapman- Kolmogorov equation. Gambler's ruin problem. |
| 4 | Bayesian Network representation. Stochastic Independence and conditional independence. Exchangeability and conditional independence. Undirected Graphical Models. Learning Graphical Models, [9] |
| Hrs | Workshops |
| 2 | Notions of combinatorial analysis in R. Properties of probability. Properties of probability and coherence principle. Algorithm for coherence checking. Probability and (decimal) odds gambling. |
| 2 | Conditional probability. Bayes rule. Bayesian learning and inference. Paradoxes in probability theory. Birthday problem and birthday attack. |
| 2 | Discrete probability. Basic discrete probability distributions in R [7]. |
| 2 | Continuous random quantities in R. Simulation |
| 2 | Random vectors. Distribution of function of random quantities. |
| 2 | Algorithms which exploit the law of large numbers and the normal approximation. |
| 2 | Discrete Markov Chains and their graphical representation in R or in Matlab. |
| 2 | Graphical representation of Baysian Network |

