UNIVERSITÀ DEGLI STUDI DI PALERMO

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| **DEPARTMENT** | Scienze Economiche, Aziendali e Statistiche |
| **ACADEMIC YEAR** | 2022/2023 |
| **MASTER'S DEGREE (MSC)** | ECONOMIC AND FINANCIAL SCIENCES |
| **SUBJECT** | MATHEMATICS FOR ECONOMICS AND FINANCE |
| **TYPE OF EDUCATIONAL ACTIVITY** | B |
| **AMBIT** | 50496-Statistico-matematico |
| **CODE** | 15507 |
| **SCIENTIFIC SECTOR(S)** | SECS-S/06 |
| **HEAD PROFESSOR(S)** | CONSIGLIO ANDREA     Professore Ordinario     Univ. di PALERMO |
| **OTHER PROFESSOR(S)** | |
| **CREDITS** | 10 |
| **INDIVIDUAL STUDY (Hrs)** | 187 |
| **COURSE ACTIVITY (Hrs)** | 63 |
| **PROPAEDEUTICAL SUBJECTS** | |
| **MUTUALIZATION** | |
| **YEAR** | 1 |
| **TERM (SEMESTER)** | 2° semester |
| **ATTENDANCE** | Not mandatory |
| **EVALUATION** | Out of 30 |
| **TEACHER OFFICE HOURS** | **CONSIGLIO ANDREA**<br>Tuesday 12:00 13:00 Edificio 13, I piano, stanza 108; Building 13, I floor, room 108<br>Thursday 12:00 13:00 Edificio 13, I piano, stanza 108; Building 13, I floor, room 108 |

**DOCENTE:** Prof. ANDREA CONSIGLIO

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| **PREREQUISITES** | Algebra. Single variable calculus. |
| **LEARNING OUTCOMES** | 1. Knowledge and understanding<br>Definition and description of the main theorems of unconstrained and constrained optimization. Ability to Identify and discuss the implications of such theorems for linear and quadratic optimization models.<br>Ability to list and describe the basic portfolio optimization models.<br>2. Applying knowledge and understanding<br>Ability to use the knowledge of specific theorems to determine the critical points of a function of several variables.<br>Implementing a GAMS model to solve a portfolio selection problem.<br>3. Making judgements<br>Analysis of a real portfolio selection problem and choice of the appropriate mathematical model.<br>4. Communication skills<br>Knowledge of the economic and financial jargon to communicate the main outcomes of a models implemented.<br>Ability to explain why a given portfolio model is preferable<br>to another, and highlight the hypotheses to be held such that the results are valid.<br>Present the results in professional way through pictures and spreadsheets.<br>5. Learning skills<br>Conduct research and analysis in the field of economics and finance using mathematical models. |
| **ASSESSMENT METHODS** | The exam is made up by two parts.<br>The first part is a written test consisting in solving mathematical problems related to the theoretical part of the course. There will be 6/8 problems which will assigned roughly the same score. For each exercise, the minimal score is assigned if the solution is just sketched or only the first steps are implemented. The total score is obtained by the sum each score.<br><br>The second part of the exam consist in implementing a portfolio model using the optimization software GAMS. The exam will be performed on a computer. A score sufficient to pass the exam will be assigned to students who prove to be able to input data, display the input data and properly recognizes the endogenous and exogenous variables of the problem.<br><br>The final mark is the arithmetic mean of the marks obtained in the two parts. |
| **EDUCATIONAL OBJECTIVES** | At the end of the course the student will be able:<br>1) To extend to the n-dimensional space the notions of single variable calculus<br>2) To define a constrained and unconstrained optimization problem<br>3) To compute the maxima and minima of constrained and unconstrained optimization problem<br>4) To distinguish between linear and nonlinear programming<br>5) To implement a GAMS model to solve an optimization problem<br>6) To represent optimal financial decision-making through optimization models. |
| **TEACHING METHODS** | Lectures, tutorial and practice skills using the software GAMS. |
| **SUGGESTED BIBLIOGRAPHY** | C. P. Simon and L. Blume. Mathematics for Economists. Norton & Company, New York, 1994. Ch. 8,9,10,12,13,14,15,16,17,18,19,21<br>A. Consiglio, S. Nielsen and S.A. Zenios. Practical Financial Optimization. Wiley Finance, 2003. All chapters. |

## SYLLABUS

| Hrs | Frontal teaching |
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| 2 | Presentation of the objectives of the course. The n-dimensional real space R^n. Representation of a point in R^n. R^n as vector space. Vector operations. The null vector. Matrix representation. |
| 2 | The dot product. Properties of the dot product. Norm in R^n. Euclidean distance in R^n. Angles in R^n and orthogonality. |
| 2 | Scalar functions of several variables. Vector functions of several variables. Linear functions. Level curves. Neighborhood in R^n. Limits of function of several variables. |
| 2 | Partial derivatives. The gradient vector and properties. Partial derivatives of higher order. The Hessian matrix. Differentiable functions in R^n. |
| 2 | Computing the partial derivatives. Graphical representation of the gradient vector. Computing the Hessian matrix. Function of class C^1 and C^2. |
| 2 | The impliciti function theorem for f: R^2->R. Tangent to a curve in R^2. Orthogonality of the gradient vector with respect to the level curves. |
| 2 | Real quadratic form. Matrix representation. Quadratic form positive and negative (semi)definite. Principal minor. Leading principal minor. Sylvester's criterion. |
| 2 | Graphical representation of a quadratic form. Application of the Sylvester's criterion. |
| 2 | Definition of maxima and minima for function of several variable. First order condition (FOC) and second order condition (SOC). Saddle points. |

# SYLLABUS

| Hrs | Frontal teaching |
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| 4 | Determine the stationary points of a functions of several variables (FOC). Assessment of the maxima, minima and saddles (SOC) for functions from R^2 to R and from R^3 to R. |
| 2 | Convex set in R^n. Convexity and concavity for function of several variables. Properties of convex function. |
| 2 | Equality constrained optimization problems. Graphical solution for function f:R^2->R. The Lagrangian function. The FOC conditions. The non-degenerate constraints qualifications. |
| 4 | Inequality constrained optimization. The necessary condition of Karush-Kuhn-Tucker. The Kuhn-Tucker conditions with non-negativity constraints. |
| 4 | The implicit function theorem for vector functions f:R^m->R^n. The Jacobian of vector functions. |
| 2 | Application of the theorem of implicit functions |
| 4 | Return of an investment. Portfolio return. Expected portfolio return and portfolio risk. The optimal portfolio selection model (aka Markowitz's Model). The efficient frontier. |
| 2 | MAD models. The absolute value in optimization: splitting the variable or splitting the equations.<br>Vector equations in GAMS |
| 2 | The profit/loss distribution of a financial position. Value-at-Risk as a measure of the market risk. Conditional VaR (CVaR). Optimization of the CVaR. |
| 4 | Utility function. Properties. The measure of risk aversion. Absolute and relative risk aversion. CARA and CRRA utility functions. Maximization model of the expected utility. |

| Hrs | Practice |
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| 2 | Solving equality constrained optimization problems. Assessing the convexity/concavity of a function of several function.<br>Using convexity/concavity of a function. |
| 2 | Solution of inequality constrained problem. |
| 2 | Introduction to GAMS. Description of the GAMS IDE. Creation of project. SET statement. Enumeration of a SET. SET as indices. ALIAS statement. SCALAR statement declaration and assignment. The DISPLAY statement. |
| 4 | Data representation. Vectors, matrices and multidimensional arrays. The PARAMETER and TABLE statement.The GDX file. Input data from a GDX file. |
| 2 | Aggregation operators: SUM, PROD, ORD, CARD, SMAX, SMIN. The $-statement<br>The VARIABLE statement. The EQUATION statement. Scalar and vector equations. The MODEL statement. The SOLVE statement. Linear (LP) and non-linear (NLP) models. |
| 4 | Implementation of the Markowitz model. Short sales. Constrain total short sales. Constraint investment on set of assets. Build the efficient frontier. The LOOP statement. |
| 2 | Maximization of the expected utility<br>CVaR model implementation |