



# UNIVERSITÀ DEGLI STUDI DI PALERMO

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|-------------------------|------------------------------------------------------------------------------------------|
| DEPARTMENT              | Ingegneria                                                                               |
| ACADEMIC YEAR           | 2022/2023                                                                                |
| BACHELOR'S DEGREE (BSC) | ENERGY ENGINEERING AND RENEWABLE ENERGIES                                                |
| INTEGRATED COURSE       | MATHEMATICAL ANALYSIS - INTEGRATED COURSE                                                |
| CODE                    | 19109                                                                                    |
| MODULES                 | Yes                                                                                      |
| NUMBER OF MODULES       | 2                                                                                        |
| SCIENTIFIC SECTOR(S)    | MAT/05                                                                                   |
| HEAD PROFESSOR(S)       | BONGIORNO<br>DONATELLA<br>Professore Associato<br>Univ. di PALERMO                       |
| OTHER PROFESSOR(S)      | BONGIORNO<br>DONATELLA<br>Professore Associato<br>Univ. di PALERMO                       |
| CREDITS                 | 12                                                                                       |
| PROPAEDEUTICAL SUBJECTS |                                                                                          |
| MUTUALIZATION           |                                                                                          |
| YEAR                    | 1                                                                                        |
| TERM (SEMESTER)         | Annual                                                                                   |
| ATTENDANCE              | Not mandatory                                                                            |
| EVALUATION              | Out of 30                                                                                |
| TEACHER OFFICE HOURS    | <b>BONGIORNO<br/>DONATELLA</b><br>Wednesday 15:00 - 17:00    piattaforma Microsoft Teams |

**DOCENTE:** Prof.ssa DONATELLA BONGIORNO

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| <b>PREREQUISITES</b>      | The student must know the basic notions of algebra and of analytical geometry, therefore he must be able to solve an equation as well as an inequality of the first and of the second order, he must also know how to solve a system of two equations with two variables and how to calculate the distance between two points, how to find the equation of a line when two points P and Q are assigned or when it is known that the line passes through an assigned point P and it is parallel or perpendicular to an assigned line r. The student has to know the trigonometry too.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| <b>LEARNING OUTCOMES</b>  | <p>Knowledge of: The student, at the end of the course, will have developed a knowledge of the fundamental elements of differential and integral calculus both for real functions of one variables that for real functions of two variables. Moreover, the student will have developed a knowledge of some elements of ordinary differential equations. In particular he will be able to understand the main concepts of calculus such as those of limit, continuity and derivability as those of definite and indefinite integral.</p> <p>Comprehension of: At the end of the course, the student will be able to know and to apply the basic notion of calculus in the solution of differential problem.</p> <p>Ability to: The student will be able to apply calculus to solve practical problems.</p> <p>All the objectives described above will be achieved by theoretical lessons and exercises moreover they will be verified in the written test.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <b>ASSESSMENT METHODS</b> | <p>The student will be evaluated through a written test of two hours long. The oral test is, mainly, optional. The written test proposed to the student will consists of six exercises, three of them will be, mainly, focus on the differential and integral calculus of a real function of one real variable, mainly covered in the first part of the course; while the remaining ones will be focus on a resolution of a Cauchy problem of some ordinary differential equations and on exercises about the differential and the integral calculus of real functions of two real variables, covered in the second part of the course. The assessment is carried out of thirty. At each exercise done correctly you will be awarded points that vary in proportion to the complexity of the year awarded. The maximum score is equal to 30/30 and will be achieved by the student who can correctly solve the six proposed exercises, showing an excellent knowledge of the topics covered and an excellent mathematical language properties. Praise can only be obtained by taking an additional oral exam in which the student must demonstrate that he is able to present the main theorems of Mathematical Analysis with perfect mathematical rigor. Moreover, the students that obtain a score between 26/30 and 29/30 will be those that have a good knowledge of the topics covered and a discrete mathematical language, the students that obtain a score between 23/30 and 25/30 will be those that have more than sufficient knowledge of the topics covered but those that have not so good mathematical language, the students that obtain a score between 22/30 and 19/30 are those that have a full enough knowledge of the topics covered but those that have a worst mathematical language. Finally, to pass the exam the minimum score is 18/30 and the students that obtain such a score are those whose knowledge of the topics covered is sufficient. Moreover, to help as much as possible the students, when the score is between 15/30 and 17/30, it is possible to do an additive little exam in order to arrive to the minimum possible score to pass the exam that it is 18/30. The students that during the written test have obtained a score lower than 15/30 are insufficient and their exam is finished. Moreover, during the period January-February will be carried out an ongoing test that allows the students to evaluate themselves. The students who decide to be evaluated by the teacher (having self-assessed at least sufficiently), at the end of the course, during the period June-July, will be obligated to do a reduced written test, containing the only topics covered in the second module. Finally, the score of such a student will be the sum of the score obtained during the ongoing test and that obtained during the second reduced test.</p> |
| <b>TEACHING METHODS</b>   | The training course consists of lectures in the classroom that are supported by a suitable teaching material provided by the teacher in a paper or in an electronic formats. To stimulate the attention and the interest of students it will be proposed some verification exercises.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |

**MODULE**  
**MATHEMATICAL ANALYSIS - MODULE 1**

*Prof.ssa DONATELLA BONGIORNO*

**SUGGESTED BIBLIOGRAPHY**

M. Bertsch, R. Dal Passo, L. Giacomelli -Analisi matematica- MACGRAW-HILL; ISBN 978-88-386-6281-2  
P. Marcellini; C. Sbordone - Esercitazioni di Analisi Matematica Uno, Prima e Seconda parte - Liguori Editore; ISBN 88-207-1684-4  
S. Salsa, A. Squellati, - Esercizi di Analisi Matematica - Volume 1- Zanichelli; ISBN978-88-08-22488-0  
M. Bramanti, D. C. Pagani, S. Salsa - Analisi Matematica 1- Zanichelli; ISBN 978-88-08-06485-1

|                               |                                            |
|-------------------------------|--------------------------------------------|
| <b>AMBIT</b>                  | 50292-Matematica, informatica e statistica |
| <b>INDIVIDUAL STUDY (Hrs)</b> | 96                                         |
| <b>COURSE ACTIVITY (Hrs)</b>  | 54                                         |

**EDUCATIONAL OBJECTIVES OF THE MODULE**

The learning objectives of the course are those of reasoning and logical deduction, to provide basic preparation to the engineering student.

**SYLLABUS**

| <b>Hrs</b> | <b>Frontal teaching</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 3          | Elementary set theory. The union, the intersection, the difference between two sets. Numerical sets: the set of natural number (the factorial, the principle of Mathematical induction), the set of integer number, the set of rational number, the set of real number. Absolute value. The upper bound and the lower bound of an ordered set; the maximum and minimum of an ordered set; the supremum and the infimum of an ordered set. The completeness axiom. The $n^{\text{th}}$ root of a real number.                                                                                                                |
| 3          | Complex numbers. Adding and multiplying of a complex number. The conjugates, the complex plane. Division of two complex number. Multiplying by the conjugate. The trigonometric formula of a complex number. The De Moivre formula. The $n^{\text{th}}$ root of a complex number. The fundamental theorem of algebra.                                                                                                                                                                                                                                                                                                       |
| 3          | Real functions. Elementary functions: the constant function, the linear function, the identity function, the quadratic function, the polynomial function, the signum function. Domain and range. Injective and surjective functions. The inverse function. Composition of functions. Examples and property.                                                                                                                                                                                                                                                                                                                 |
| 4          | An Introduction to Limits. The unicity of the limits. Limits on the right and limits on the left. Definition of continuity. Continuity at a point. Limits of a continuous functions. Asymptotes. Monotone functions and their rules for limits. Limits of composition functions. Change of variables.                                                                                                                                                                                                                                                                                                                       |
| 3          | Discontinuities: Removable, jump discontinuities. Continuity on an interval. The Intermediate Value Theorem. The Weierstrass theorem. The zero theorem.                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| 4          | The Definition of the Derivative. Geometrical and kinematical interpretation of the derivatives. Examples. The derivative of some elementary functions; Derivative of a constant function, Derivatives of polynomials, Derivatives of Trigonometric Functions, Derivatives of Exponential functions, Derivative of logarithmic functions. Differentiation rules. Derivatives on the right and derivatives on the left. Classifications of the points of not derivative for a function. The derivative of the inverse function. The derivative of the composition function. Derivative of higher order. The Convex functions |
| 4          | Extreme points. The Rolle's theorem, the Lagrange's theorem. Some consequences of the Lagrange's theorem. The notion of primitive. How to get the graph for a given function and how represent it on the Cartesian plane. The De l'Hospital's rule. The Taylor's formula and its applications.                                                                                                                                                                                                                                                                                                                              |
| 2          | The Riemann integral and its rules. Some class of functions that are Riemann integrable. The mean value theorem. The fundamental theorem of calculus                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <b>Hrs</b> | <b>Practice</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| 28         | Exercises about real and complex numbers. Exercises about real function. Exercises about the computation of limits even with the use of the Taylor's formula or the De Hospital formula. Exercises about continuous functions. Some integration rules.                                                                                                                                                                                                                                                                                                                                                                      |

## MODULE MATHEMATICAL ANALYSIS - MODULE 2

*Prof.ssa DONATELLA BONGIORNO*

### SUGGESTED BIBLIOGRAPHY

M. Bramanti, C.D. Pagani, S. Salsa "Analisi Matematica 2", ZANICHELLI; ISBN 978-88-08-12281-0  
M. Bertsch, R. Dal Passo, L. Giacomelli "Analisi matematica" MACGRAW-HILL; ISBN 978-88-386-6281-2  
P. Marcellini; C. Sbordone - Esercitazione di Analisi Matematica Due. Prima e Seconda parte. - Liguori Editore; ISBN 978-88-08-19145-8  
S. Salsa, A. Squellati, - Esercizi di Matematica 2 - Zanichelli; ISBN 978-88-08-22958-8

|                               |                                            |
|-------------------------------|--------------------------------------------|
| <b>AMBIT</b>                  | 50292-Matematica, informatica e statistica |
| <b>INDIVIDUAL STUDY (Hrs)</b> | 96                                         |
| <b>COURSE ACTIVITY (Hrs)</b>  | 54                                         |

### EDUCATIONAL OBJECTIVES OF THE MODULE

The learning objectives of the course are those of reasoning and logical deduction, to provide basic preparation to the engineering student.

## SYLLABUS

| Hrs | Frontal teaching                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4   | The improper integral. definition and the convergent criteria.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 4   | Definition of numerical series. Examples: Mengoli series, geometric series, harmonic series, generalized harmonic series. Sum of a series. Necessary condition but not sufficient for the convergence of a series. Series with non-negative terms. Convergence criteria: the criterion of comparison, the asymptotic comparison, the criterion of the nth root, the integral criterion, the condensation Cauchy criterion.                                                                                                                                                                                                                                            |
| 5   | First order differential equations. Differential equation with separable variables: Theorem of existence and uniqueness for the Cauchy problem. Solutions "big" and "small". Techniques for the computation of differential equations with separable variables. First order linear differential equations. The method of variation of an arbitrary constant. Linear differential equations of higher order. The characteristic polynomial. General solution of the homogeneous linear differential equation with constant coefficients. Linear differential equations with constant coefficients nonomogee. Method of similarity and method of variation of constants |
| 5   | Partial derivatives. $C^k$ class functions. The Jacobian, the gradient and the differential. Linear approximation. Relationship between differentiability, partial derivatives and $C^1$ class functions. The gradient to estimate the maximum growth of a function. The directional derivatives. Maximum and minimum for functions from $R^2$ to $R$ .. Stationary points and the Fermat's theorem . The Hessian Matrix.                                                                                                                                                                                                                                             |
| 4   | Techniques for the study of maxima and minima of a function restricted to a curve of the plan: reducing the number of variables; parameterization of the curve with a parameter $t$ ; of Lagrange multipliers method                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 2   | Definition of double integral for $f: A \rightarrow R$ continues, with $A$ regular domain of the plane. Riemann sums. Properties of the double integral (linearity, additivity, monotony ...) and geometric interpretation. Theorem of the integral average.                                                                                                                                                                                                                                                                                                                                                                                                          |
| 4   | Reduction formulas for double integrals. Change of variables in double integrals. Double integrals in polar coordinates.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| Hrs | Practice                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 5   | Exercises about differentiation of a given function of two real variable. Exercises about the calculation of the diretional derivatives for a given function of two real variable. Determination of the nature of the extrem points with the help of the Hessian Matrix or without.                                                                                                                                                                                                                                                                                                                                                                                   |
| 21  | Exercises about differential equation.<br>Exercises about differentiation of a given function of two real variable. Exercises about the calculation of the diretional derivatives for a given function of two real variable. Determination of the nature of the extrem points with the help of the Hessian Matrix or without. Exercises regarding the calculation of some integrals of real functions of two real variables of normal domains of the plan with or without change of variables.                                                                                                                                                                        |