UNIVERSITÀ DEGLI STUDI DI PALERMO

DEPARTMENT	Matematica e Informatica
ACADEMIC YEAR	2019/2020
BACHELOR'S DEGREE (BSC)	MATHEMATICS
INTEGRATED COURSE	DYNAMIC SYSTEMS WITH LABORATORY
CODE	11081
MODULES	Yes
NUMBER OF MODULES	2
SCIENTIFIC SECTOR(S)	MAT/07
HEAD PROFESSOR(S)	LOMBARDO MARIA CARMELA — Professore Ordinario — Univ. di PALERMO
OTHER PROFESSOR(S)	GAMBINO GAETANA — Professore Associato — Univ. di PALERMO LOMBARDO MARIA CARMELA — Professore Ordinario — Univ. di PALERMO
CREDITS	12
PROPAEDEUTICAL SUBJECTS	
MUTUALIZATION	
YEAR	2
TERM (SEMESTER)	Annual
ATTENDANCE	Not mandatory
EVALUATION	Out of 30
TEACHER OFFICE HOURS	**GAMBINO GAETANA** Thursday 11:00 13:00 Ufficio del docente: Stanza n. 216, Secondo Piano, Dipartimento di Matematica e Informatica, Via Archirafi 34 **LOMBARDO MARIA CARMELA** Tuesday 11:30 12:30 Studio del docente, presso il Dipartimento di Matematice e Informatica. Per confermare data, orario e sede del ricevimento e' necessario contattare il docente tramite e-mail.

DOCENTE: Prof.ssa MARIA CARMELA LOMBARDO

PREREQUISITES	Real functions of one variable. Elementary functions. Limits, continuity and differentiability. Taylor series. Vectorial spaces. Endomorphisms. Matrix calculus. Eigenvalues and eigenvectors. Diagonalization.
LEARNING OUTCOMES	Knowledge and Understanding: Knowledge of the main concepts of Dynamical Systems: equilibria, stability, periodic orbits, limit cycles, bifurcations of one parameter dynamical systems. Being able to describe and model simple phenomena from real life. The students will acquire knowledge and understanding through participation to the frontal lessons, discussion hours, laboratory sessions and individual study. Applying knowledge and understanding: The students will be able to formalize in mathematical terms moderatly difficult problems and to extract qualitative behaviours from quantitative data. In particular, they will acquire the following competence: stability analysis of equilibria through linearization and the Lyapunov' theorem, Poincare-Bendixon theorem for the existence of the limit cycle, normal reduction form close to bifurcation, numerically computed bifurcation diagram, asymptotic expansion in presence of small parameters, numerical simulations of finite dimensional dynamical systems. Making judgments: Being able to formulate an evolutive mathematical model and to determine its limits of applicability, also making comparisons between the numerical solutions and the experimental data. Being able to generalize the model to more complex and realistic situations. To understand mathematical models describing real phenomena in different scientific contexts. Group experience during laboratory and discussion sessions. Communication skills : To own suitable competence and communication skills. In particular, to be able to explain elementary models coming from physics, ecology and biology to high school students, discuss their solutions and simulate the corresponding dynamical behaviours. Being able to discuss with experts of other disciplines formalizing in mathematical terms problems relevant to industrial or financial applications. Learning skills: Comprehension of simple scientific papers (e.g. papers appeared in the Education section of SIAM Review) about modeling of physical and biomathematical problems, understanding the theoretical and numerical analysis of the proposed models.
ASSESSMENT METHODS	The final verification aims to estimate: the knowledge and the understanding of the student about the contents of the course; the competence of the student to apply this knowledge and understanding; if the student owns autonomy of judgments and suitable communication and learning skills. For midterm written tests will be administered for self evaluation, in each of which the resolution of three exercises will be requested. The evaluation of each midterm exam will be out of 30.The final verification consists of a written test, a laboratory exam and an oral examination. In the written test the resolution of four exercises is demanded. The exercises refer to all the material covered by the program and are consistent to the examples and the discussion hours developed during the course. The evaluation of the written exam will be out of 30. The students who have attended at least the 75% of the lectures can choose to take 4 midterm written tests instead of the final written exam. In this case, the final score of the written exam is computed as the arithmetic mean of the 4 midterm scores. During the laboratory exam the student will be asked to simulate a dynamical system or reproduce a numerical bifurcation diagram, consistent to the examples and the discussion hours developed during the course. The evaluation of the laboratory exam will be out of 30. During the oral examination the student should correctly answer to two/three questions based on all the contents of the course. Moreover, the student should give a critical explanation of the exercises' resolution proposed in the written test. The evalutation of the oral exam will be formulated out of 30. The final evaluation will be based on both the written test, on the laboratory exam and the oral examination and will be computed as the arithmetic mean of the written exam (or midterm mean score), the laboratory exam score and the oral exam scores. It will be scaled according to the following conditions: a) does not possess an acceptable knowledge of the contents of the presented topics (no sufficient); b) minimal base knowledge of the contents of the course and of the technical language, minimal ability to independently apply the acquired knowledge (18-20); c) not have full mastery of the main contents of the course but possesses knowledge, satisfactory property of language, exiguous ability to independently apply the acquired knowledge (21-23); d)knowledge of base treated contents, discrete property of language, with acceptable ability to independently apply the competence to solve the proposed problems (24-25); e) good mastery of the contents of the course, very good property of language, good competence in problem-solving (26-29); f) optimal knowledge of the contents of the course, optimal property of language,

	very good analytic abilities and competence in problem solving (30-30 with honors).
TEACHING METHODS	The course is organized in frontal lessons and discussion hours. During the frontal lessons all the contents of the course will be rigorously presented and analyzed. Through the exercises the students will acquire greater understanding of the presented topics. In particular, partial written tests will be proposed to prepare the students to the final verification and whose positive evaluation may substitute the final written exam.

MODULE
BIFURCATIONS AND SINGULAR PERTURBATIONS

Prof.ssa GAETANA GAMBINO

SUGGESTED BIBLIOGRAPHY

Testo di riferimento (Textbook):
S.H. Strogatz, Nonlinear Dynamics and Chaos, Westview Press, 2000.

Testi di consultazione (Reference books):
R. Haberman, Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow (Classics in Applied Mathematics) , SIAM, 1998.
J.D.Murray, Mathematical Biology, 3rd edition, Springer-Verlag, 2002.
J. Hale and H. Kocak, Dynamics and Bifurcations, Springer-Verlag, 1991.
F. Brauer, C.Castillo Chavez, Mathematical models in Population Biology and Epidemiology, Springer, 2000.
K. Chen, P. Giblin, A. Irving Mathematical explorations with MATLAB, Cambridge University Press, 1999.

AMBIT	50195-Formazione Modellistico-Applicativa
INDIVIDUAL STUDY (Hrs)	94
COURSE ACTIVITY (Hrs)	56

EDUCATIONAL OBJECTIVES OF THE MODULE

Introduction of the fundamental elements for the qualitative analysis of a finite dimensional dynamical system in R^2 and R^n systems, also in the presence of parameters. In particular:
1) Construction and analysis of the bifurcation diagram in the presence of parameters;
2) Poincaré-Bendixon Theorem.

SYLLABUS

Hrs	Frontal teaching
6	Bifurcation theory: attractors of a dynamical system. Bifurcations of one-parameter 1D dynamical systems: fold bifurcation, transcritical bifurcation, pitchfork bifurcation. Imperfect bifurcations.
4	Bifurcations of a 2D dynamical system in the presence of a zero eigenvalue. Central manifolds.
6	Omega-limit and alpha-limit sets. Limit cycles. Conditions for non existence of closed orbit: Dulac theorem. Lyapunov Theorem.
6	Gradient systems. Stability of limit cycles. Poincaré-Bendixon theorem. Conservative systems. Hamiltonian systems.
6	Asymptotic analysis: definitions and examples. Regular perturbation problems. Singularly perturbed problems. Initial layer. The method of multiple scales. Error estimate. Enzyme reaction kinetics. Law mass action. The Michaelis-Menten model. The pseudo-steady-state hypothesis. Asymptotic analysis of the model.
4	Slow-fast systems: definition. Systems with different time scales. Phase-plane qualitative analysis. Conditions for the existence of a limit cycle. The Van der Pol oscillator: estimate of the oscillation period.

Hrs	Practice
2	Numerical investigations of hysterestic phenomena in one dimensional dynamical systems depending on one parameter
6	Computing numerical bifurcation diagrams of one parameter dynamical systems
6	Qualitative and numerical analysis of systems supporting oscillations.
3	Qualitative analysis of gradient and hamiltonian systems.
4	Discussion and exercises: Qualitative and numerical analysis of systems with Michaelis-Menten-type kinetics. Qualitative analysis of systems with small parameters.
3	Qualitative analysis of slow-fast systems and estimate of the oscillation period.

MODULE MAPS, EQUILIBRIUMS, STABILITY *Prof.ssa MARIA CARMELA LOMBARDO*	
SUGGESTED BIBLIOGRAPHY	
Testo di riferimento (Textbook): Salinelli E., Tomarelli F., Modelli dinamici discreti, Springer, 2005. Testi di consultazione (Reference books): Mooney-R. Swift, A Course in Mathematical Modeling, The Mathematical Association of America, 1999. S.H. Strogatz, Nonlinear Dynamics and Chaos, Westview Press, 2000.	
AMBIT	50197-Formazione Matematica di base
INDIVIDUAL STUDY (Hrs)	94
COURSE ACTIVITY (Hrs)	56
EDUCATIONAL OBJECTIVES OF THE MODULE	
The main aim of the course is introducing the basic concepts and ideas in order to perform the qualitative analysis of a finite dimensional dynamical system and to study its solutions in the phase plane. In particular: 1) Linearization in the neighbourhood of an equilibrium point and stability analysis for maps (discrete dynamical systems); 2) Linearization in the neighbourhood of an equilibrium point and stability analysis for continuous dynamical systems; 3) Global phase portrait. A further educational objective is introducing the student to the basic problems arising in the mathematical modeling: for this purpose simple mathematical models of physical and biomathematical interest will be formulate and theoretically and numerically analyzed .	

SYLLABUS

Hrs	Frontal teaching
6	An overview of the course. Introduction to the theory of dynamical system, definition of a discrete dynamical system (map) and its solution, linear and nonlinear system, phase space, orbits, equilibria, stability. Cobweb method.
6	Linear maps of order one: solutions, equilibria and stability. Linear maps of order m>1: solutions and their asymptotic behaviours. Nonlinear maps. Linearization theorem
6	Continuous dynamical systems: definition and solutions. Linear and nonlinear systems, phase space, equilibria and stability. Cauchy Theorem. Continuous dependence on the initial data. Hartmann-Grossmann Theorem.
4	Classification of equilibrium points: attractive and repulsive node, saddle, centers, spirals. Linear continuous dynamical systems: solutions and phase portraits.
5	Evolution equation in a one dimensional phase space: Malthus equation, logistic equation and its derivation, Gomperz law, compensation and depensation models, Allee effect . Population growth in presence of a constant/linear tax rate hunting term. Hysteresis phenomenon: spruce budworm. Dynamical system with delay: logistic equation with a delayed growth rate, analysis of the oscillation period.
5	Evolution systems in a multi-dimensional phase space: interacting species (competition, symbiosis, predation). Predator-prey models: global phase portrait of the Lotka-Volterra model.

Hrs	Practice
4	Qualitative analysis and numerical simulations of some linear maps arising in applied mathematics context.
4	Qualitative analysis and numerical simulations of some nonlinear maps arising in applied mathematics context.
6	Qualitative analysis and numerical simulations of some continuous linear dynamical systems arising in applied mathematical context (harmonic oscillator: simple, damped and forced).
5	Qualitative analysis and numerical simulations of nonlinear continuous dynamical sistems in one dimensional phase space
5	Qualitative analysis and numerical simulations of nonlinear continuous dynamical sistems in multi-dimansional phase space