## UNIVERSITÀ DEGLI STUDI DI PALERMO



| PREREQUISITES | Combinatorics. <br> Analytic geometry. <br> Linear systems. <br> Set theory. <br> Sequences and series. <br> Differential and integral calculus. <br> Complex numbers. |
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| LEARNING OUTCOMES | 1) KNOWLEDGE AND UNDERSTANDING <br> The student should know the following topics: <br> - Elements of logic and combinatorics; <br> - Various concepts od probability <br> - Setting of Coherence <br> - Elementary Probability; <br> - Events and conditional events; <br> - Random quantities and probability distributions; <br> - Summary statistics; <br> - Classical problems of probability. <br> - Random vectors, joint and marginal probability distributions; <br> - Functions of random quantities and of random vectors; <br> - Different relations among random quantities; <br> - Some limit theorems. <br> 2) APPLYING KNOWLEDGE AND UNDERSTANDING <br> The student should know how to exploit probabilistic tools to reasoning under uncertainty. In particular, the student should be able to: <br> - Describe the uncertainty; <br> - Check the coherence of a probability assessment on an arbitrary finite family of events; <br> - Apply Bayes' rule and compound probability theorem; <br> - Solve classical problems of probability; <br> - Select the suitable probability distribution to the describe some standard random problems; <br> - Apply limit theorems; <br> - Obtain marginal probability distributions from joint probability distributions. <br> - Solve problems involving functions of random quantities. <br> 3) MAKING JUDGEMENTS <br> Being able to motivate the right probabilistic tools and models which can be used to solve problem under partial knowledge. <br> 4)COMMUNICATION. <br> The student should know how illustrate in a clear and coherent way the description and analysis of a random problem to both expert and non-expert people. <br> 5) LIFELONG LEARNING SKILLS. <br> Being able to exploit the power of self-discovery, exploration, learning and mastery. In particular the student should be able to deep some well-known notions in probability theory and to discover new notions by studying from other research books or from research articles. |
| ASSESSMENT METHODS | The exam is composed by a written test (or two intermediated written tests) followed by an oral test. <br> Written tests <br> The written test (of three hours) will be composed of 4-6 exercises: 2-3 <br> concerning topics of the first half of the course (e.g., discrete probabilities, summary statistics, ...) and <br> 2-3 concerning topics of the second half of the course (e.g., continuous random quantities, random vector, limit theorems, characteristic function). <br> At each exercise corrected solved will be given the same score. <br> The evaluation of the written test will be made by means of a 30-point scale: non passing from 0 to 17; passing from 18 to 30. <br> Candidates who do not pass the written test are encouraged to repeat the written test. <br> Intermediated written tests. <br> The first intermediated written test (of one hour and a half) will be composed of 2-3 exercises <br> concerning topics of the first half of the course. <br> The second intermediated written test (of one hour and a half) will be composed of 2-3 exercises <br> concerning topics of the second half of the course. <br> At each exercise corrected solved will be given the same score. <br> The evaluation of the intermediated written test will be made by means of a 30- <br> point scale: non passing from 0 to 17 ; passing from 18 to 30 . Those who pass |


|  | both intermediated written tests will avoid the written test and will receive as score of the written test the average of the two intermediate scores. Some solved tests are available at http://www.unipa.it/sanfilippo . The written tests are designed in order to evaluate the degree of knowledge and the ability of the candidate to solve problems similar to those illustrated during the lectures and during practicals. <br> In particular, through them it will possible to measure the ability of the candidate in order to: <br> formalize a random phenomenon; <br> motivate the choice of a probabilistic model and of some probability distributions; provide summary statistics; <br> understand conditionalization; <br> apply or re-elaborate theoretical results; apply limit results; <br> exploit standard normal table; <br> apply mathematical tools to solve random problems. <br> Oral Test <br> By starting from a deepening on the written test, the oral test, through the formulation some questions, will be used to <br> evaluate the degree of knowledge of the student in some theoretical results and in the use of technical language. The score of the oral test will be a number usually belonging to the set $\{-4,-3,-2,-1,0,1,2,3,4\}$. The score of the oral test will be used to finalize and / or improve the grade obtained in the written tests. Finale grade <br> By setting S="the score of the written test" and O="the score of the oral test", the final grade V usually will be $\mathrm{V}=\mathrm{min}(\mathrm{S}+\mathrm{O}, 30)$, if $\mathrm{S}+\mathrm{O}>0, \mathrm{~V}=0$ otherwise. the average between the score of the oral test and the score of the written test. The student pass the exam if the final grade is greater or equal than 18/30 The minimum grade $18 / 30$ in order to pass the exam will be give to the student that has understood the knowledge base of the course, that is able to do some links among the different topics and that is able to use technical language. The maximum grade $30 / 30$ e lode will be given to the student that fully reached the learning outcomes. and with S+O greater than 30. |
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| EDUCATIONAL OBJECTIVES | Being able to describe and represent some theoretical and real random problems through suitable probabilistic models. In particular, the student should be able to: motivate the choice of some probability distributions; provide the summary statistics; properly assess degree of believe to (conditional or simple) events; update probability; exploit theoretical results for the study and analysis of function random vectors. Moreover, by means of the basic notions the student should know how to solve some "paradoxes" of probability. |
| TEACHING METHODS | Lectures and practicals. Usually, theory will be explained in the lecture and then applied and tested in the practical class. |
| SUGGESTED BIBLIOGRAPHY | Testi consigliati (Bibliography) <br> -[1] Sheldon Ross; Calcolo delle Probabilita' 3a ed.; Apogeo, 2013; (English version: A first course in Probability, 8th edition, Pearson) <br> -[2] Romano Scozzafava; Incertezza e Probabilita'; Zanichelli, 2003; <br> Approfondimenti (Further bibliography) <br> -[3] Paolo Baldi; Calcolo delle Probabilita'; McGraw-Hill, 2011; <br> -[4] Bruno de Finetti; Teoria delle Probabilita'; Giuffre, 2005 (ristampa); <br> (English version of the book: Theory of Probability: A critical introductory treatment, Wiley, 2017) <br> -[5] Luciano Daboni; Calcolo delle Probabilita' ed Elementi di Statistica; Utet. <br> -[6] Giorgio Dall'Aglio; Calcolo delle Probabilita'; Zanichelli, 2001; <br> Materiale didattico curato dal docente disponibile a inizio corso. <br> Esercizi svolti relativi a precedenti prove di esame. <br> (Notes and exercises provided by the Teacher) <br> Social <br> Gruppo facebook (Facebook group): https://www.facebook.com/groups/ cdp.dmi.unipa |

## SYLLABUS

| Hrs | Frontal teaching |
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| 2 | Historical note. Classical problems ([6]). Events, logical operations and relations ([2]). De Morgan's Law. Basic <br> notions on combinatorial Analysis. Power set. Finite partition. Properties of probability. |
| 2 | The main Interpretations of probability: classical, axiomatic ([3]), frequency, subjective ([2]). Betting criterion <br> and coherence principle ([2]). Probability and measure theory ([3]). |
| 2 | Conditional events and conditional probability [2]. Properties of conditional probability [1]. <br> Compound probability theorem. Disintegration law. Bayes' rule. Stochastic independence of events.. |
| 5 | Simple Random quantities [2]. Urn problems. Binomial distribution. Hypergeometric distribution. Mixture of <br> distributions [2]. Expected value and variance. Exchangeability. |

## SYLLABUS

| Hrs | Frontal teaching |
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| 4 | Discrete probability. Properties of expectation and variance. Cumulative distribution and discrete random <br> quantities. <br> Poisson and geometric distribution. Property of memoryless of the geometric distribution. Pascal Distribution <br> [2]. Markov's inequality. Chebyshev's inequality. |
| 5 | Infinity and probability. Continuous random quantities. Density [2,3], cumulative distribution function, <br> expectation, variance. Property of continuous random quantities. Some continuous distributions: Uniform, <br> Exponential, Normal, Gamma, Beta, Chi-square, Laplace, Power law, etc. Hazard rate function [1,2]. Survival <br> function. Memoryless proprerty of exponential distribution. |
| 6 | Discrete and continuous random vectors. Cdf, joint distribution functions, marginal distributions, conditional <br> distributions. Independence. Property of expectation. Covariance. Covariance matrix. Linear regression. <br> Normal distribution. Distribution of function of random quantities [see also 3]. |
| 3 | Characteristic function [2,3,6] (or in alternative moment generating function [1]) and properties. Sum of random <br> quantities. Convolution operator. General Chi-square distribution. |
| 3 | Convergences of random quantities (see also [3,6]). Limits theorem. Legge dei grandi numeri. Central limit <br> theorem. Normal approximation of binomial distribution. |
| Hrs | Practice |
| 2 | Preliminaries notion of combinatorial analysis. Logical operations among events. Properties of probability. |
| 4 | Constituents associated with a family of n events. Decomposition formula. Events logically independent. <br> Properties of probability and coherence principle. Coherence checking. Probability and (decimal) odds <br> gambling. |
| 3 | Conditional probability. Bayes rule. Paradoxes in probability theory. |
| 3 | Discrete probability |
| 4 | Continuous random quantities. Summary statistics. Survival analysis. |
| 4 | Random vectors. Distribution of function of random quantities. |
| Normal distribution. Limits Theorem. Characteristic function (or generating moment function) |  |
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