STRUTTURA	Scuola Politecnica – Dipaerimento SEAS
ANNO ACCADEMICO	2014/2015
CORSO DI LAUREA MAGISTRALE	Scienze Statistiche (LM82)
INSEGNAMENTO	Stochastic processes
TIPO DI ATTIVITÀ	Caratterizzante
AMBITO DISCIPLINARE	Matematico applicato
CODICE INSEGNAMENTO	16439
ARTICOLAZIONE IN MODULI	NO
SETTORI SCIENTIFICO DISCIPLINARI	MAT/06
DOCENTE RESPONSABILE	Da definire
CFU	8
NUMERO DI ORE RISERVATE ALLO STUDIO PERSONALE	140
NUMERO DI ORE RISERVATE ALLE ATTIVITÀ DIDATTICHE ASSISTITE	60 (36 lezione frontale + 24 esercitazione/lab.)
PROPEDEUTICITÀ	nessuna
ANNO DI CORSO	Primo
SEDE DI SVOLGIMENTO DELLE LEZIONI	Consultare il sito della scuola politecnica politecnica.unipa.it
ORGANIZZAZIONE DELLA DIDATTICA	Lezioni frontali, Esercitazioni in aula
MODALITÀ DI FREQUENZA	Facoltativa
METODI DI VALUTAZIONE	Prova orale
TIPO DI VALUTAZIONE	Voto in trentesimi
PERIODO DELLE LEZIONI	Consultare il sito della scuola politecnica politecnica.unipa.it
CALENDARIO DELLE ATTIVITÀ DIDATTICHE	Consultare il sito della scuola politecnica politecnica.unipa.it
ORARIO DI RICEVIMENTO DEGLI STUDENTI	Consultare la pagina personale del docente

EXPECTED LEARNING OUTCOMES

Knowledge and understanding

At the end of the course, students should show knowledge and comprehension:

- -of probability theory and measure theory;
- -of characteristic funtions, and several types of convergence of random sequences.
- -of the definition of a stochastic process (S.P.)
- -of classification of S.P.
- -of Random walk and the Gambler's ruin problem;
- -of discrete Markov chains and theory of state classification and theory of equilibrium;
- -of time continuous S.P.;
- -of Poisson process, Renewal and Renewal-Reward process;
- -of continuous Markov chains and queues;
- -of Martingales, strategies and stopping time;
- -of arbitrage, reproducing strategy, self-financing strategy and hedging
- -of Binomial market model, Black-Scholes model and option evaluation

Applying knowledge and understanding

Students should become able to apply their knowledge and comprehension to tackle problems of uncertainty by means of suitable stochastic models. Specifically, students should be capable to:

- -classify a stochastic process;
- -interpret different forms of stochastic dependencies;
- -describe a time and space dependent random process with a suitable stochastic process;
- -apply the aquired knowledge to solve stochastic problems proper of other subjects (financial matematics, advanced statistics, queue theory)

Making judgements

Students should become able to recognise the significant elements of a problem of uncertainty, thereby assessing the probabilistic tools used to tackle the problem.

Communication skills

Students should become able to communicate the analysis of a random phenomenon to specilized and unspecialized audiences.

Learning skills

Studends should have developed adequate learning ability to undertake subsequent studies with high degree of independence. Specifically, students should be able to:

- examine national and international scientific literature;
- further deepen their mathematical knowledge;
- design problems and seek the relative solutions.

COURSE OBJECTIVES

This course exposes the theory and applications of stochastic processes, focussing on some of the most frequently exploited models in applied sciences.

PRELIMINARY KNOWLEDGE

Analytical geometry on plane; numeric sequences and series; differential and integral calculus, with multiple integration; probability theory.

SUGGESTED TEXTS

- Introduction to probability models, Sheldon Ross, Academic Press, 2008;
- Stochastic Processes & Models, David Stirzaker, Oxford University Press, 2005
- Financial Calculus, M. Baxter and A. Rennie, Cambridge University Press 1997;
- Probability and measure, Patrick Billingsley, Wiley, 1986;
- Probability and random processes, G. Grimmett and D. Stirzaker, Oxford Univ. Press, 2008;
- Calcolo delle Probabilità, Giorgio Dall'Aglio, Zanichelli, 2001;
- Basic Stochastic Processes, Brzezniak and Zastawniak, Springer, 1998;
- Modeling and Analysis of Stochastic Systems, V. G. Kulkarni, Chapman & Hall, 1995;
- A Course in Financial Calculus, Alison Etheridge, Cambridge University Press, 2002;
- Lecture notes.

HOURS	FRONT CLASSES
5	Review of elementary theory of probability and measure theory. Conditional expectation and its properties. Characteristic function and generating function of random variables. Convergence of random variables.
3	Definition of a Stochastic Process (S.P.). Classification of Stochastic processes. Example of S.P.: Random Walk, gambler's ruin problem.
6	Discrete time processes. Definition of Markovian process, and discrete time Markov chain (DTMC). Stochastic matrix and its properties. Chapman-Kolmogorov equation. State classification, class properties and irreducibility. First hitting time, and first passage time. Stationary distribution (SD), and reversibility. Periodicity. Ergodicity. Finite state chains. Random walk on a graph. Infinite state chain. Invariant measures.
4	Continuous time processes. Exponential variables and their properties. Counting processes. Processi di conteggio. Poisson processe. Interarrival times. Local definition of Poisson processes. Splitting, thinning and superposition. Inhomogeneous sampling.
3	Renewal process. Renewal function. Moments and their properties. First renewal theorem and elementary renewal theorem. Reward-renewal processes. Limit theorems. Applications to Markov chains (Ergodicity).
5	Continuous time Markov chains (CTMC). Markov property. Sojourn times. Memoryless property os sojourn times. CTMC characterization: underlying chain and sojourn times. Q-Matrix. Honesty and explosiveness. Chapman-Kolmogorov equation. Birth-Death processes. Elementary queue processes. Kolmogorov Forward e Backward equations. State classifications. Occupation times.

2	SD of a CTMC. Balance equation. Relation between SD of a CTMC and SD of its underlying. Convergence and ergodicity. Reversibility. Detailed balance equation.
2	Filtration and adapted process. Martingales. Discrete time and continuous time martingales. Strategies. Stopping times. Stopped martingale. Optional stopping theorem.
3	Introduction to quantitative finance. Time value of money. Financial derivatives: forward and options. Concept of arbitrage. Binomial market model. Replicating portfolio. Real probabilities and arbitrage probabilities. Portfolio hedging. Self-financing strategies. Binomial representation theorem.
3	Fundamental theorem of asset pricing. Discounted option as a martingale. Lognormal market model as limit of binomial market model. Black & Scholes formula. Vanilla option evaluation.
	TUTORIALS
2	Review of probability theory and measure theory. Conditional expectation. Characteristic and generating functions. Random sum.
2	Random walk and gambler's ruin problem.
5	Markov Chains. State classification of a DTMC. Reducible and irreducible chains, periodicity. Absorbing states. Hitting time. Reversibility. Stationary distributions.
4	Poisson processes. Examples: servers, arrival probabilities, number of births. Splitting/Superposition/Thinning. Inhomogenous sampling.
3	Application of renewal and reward-renewal processes: queueless server, car buyer model. Ispeting paradox.
4	Application of CTMC: Yule process, linear immigration model, M/M/s queue. Stationary distribution. Reversibility.
2	Example of Filtration. Example of Martingales. Application of the optional stopping time theorem: gambler's ruin problem.
2	Example of Opzions: digital options, eurpean put and call options. Hedging threes. Option evaluation with Black&Scholes formula.